M^2 = Math Mediator Lesson 43: Graphing Rational Expressions

Total Recall (Warm-up) (5 minutes approx.)

Total Recall: Exercises based on yesterday's lesson on Reducing Rational Expressions:

1. If x ≠ a; which of the following is a true statement?
   a) (x – a)/(a – x) = 1; b) (x – a)/(a – x) = -1;
   c) $(x^2 – a^2)/(x – a) = 1$ d) $(x^2 – a^2)/(x – a) = -1$ Answer: b
2. Reduce the rational expression to lowest terms:
   a. $(3bx^2)/(3bx)^2$ Answer: 1/3b
   b. $(16x^2)/(2x^2 – 4x)$ Answer: (8x)/(x – 2)

Direct Instruction: (10 minutes approx.)

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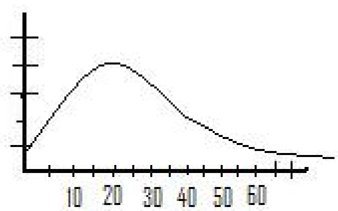
In the medical field, the expression: $\frac{5t}{0.01t^2+3.3}$; represents the concentration in micrograms of a hypothetical medicine given orally to a patient after 't' minutes in the bloodstream. The doctor uses such equations to ensure a patient has sufficient medicine and to determine at what time intervals the next dosage is given. Using a table or a graph is a useful tool to quickly prescribe the medicine.

One method is to create an x-y table of typical values. In this case, a list of typical time (we will use 't' for 'x'; and 'y' for the solution to the expression) intervals would be handy for the doctor. Ten minute intervals might be useful. Another method to display the data is with a graph. Both are shown below.

t	y
10	5(10)/(.01(100)+3.3) =11.63 mg
20	5(20)/(.01(400)+3.3) = 13.7 mg
30	5(30)/(.01(900)+3.3) = 12.2 mg
40	5(40)/(.01(1600)+3.3 = 10.36 mg
50	5(50)/(.01(2500)+3.3 = 8.83 mg



Have the students graph this expression on their calculators. Explain to them that this expression never reaches or crosses the x axis as time gets larger and larger. Ask them if the patient must be kept above 2 mg, when would the next dosage be administered? The closest answer is at 240 minutes.

Practice: (8 minutes approx.)

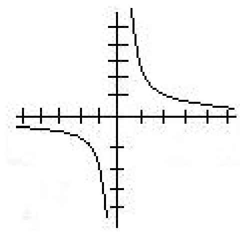
U-DO: On the graphing calculator, have students graph y = 1/x. If they don't have a calculator, have them make a table. Ask for what value of 'x' is 'y' meaningless? That would be 0. As 'x' gets extremely large, positive or negative, 'y' approaches what value? Zero is correct. As 'x' approaches 0, what values does 'y' approach? Infinity.

As 'x' values become very large, negative or positive, does 'y' ever change values from positive to negative or vice-a-versa? No. Therefore, since values approach and do not cross the y = 0 line, or the 'x' axis, that is called an "asymptote" line. The asymptote is a line that a graph approaches, but never crosses.

As ‘y’ values become very large, or ‘x’ values get closer to zero, does the graph ever cross the ‘y’ axis uninterrupted? No. The ‘y’ axis is another asymptote.

x	y
1	1
.1	10
.01	100
.001	1000

x	y
100	.01
1000	.001
10000	.0001



Direct Instruction and Practice: (10 minutes approx.)

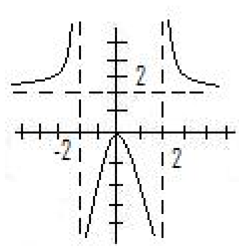
What are the asymptotes for: $y = (2x^2)/(x^2 - 4)$?

Have the students graph this rational expression on their calculators or build tables to discover what the asymptote lines are.

The asymptote lines are x = ±2 and y = 2. There are three asymptote lines.

x	y
-3	3.6
-2	undef.
-1	-.667
0	0

x	y
1	-.667
2	undef.
3	3.6
4	2.667

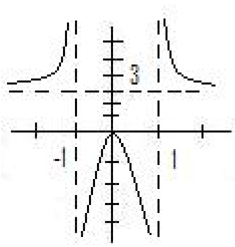


Notice the ‘x’ value of 0 produces a ‘y’ value of 0. This was called a ‘zero’ from a previous lesson. The ‘x’ value that makes the expression 0 is called a ‘zero’.

Practice (10 minutes approx.)

U-DO: Graph the following and list the asymptote lines, if any.

$y = \frac{3x^2}{x^2 - 1}$ The asymptote lines are: x = ±1 and y = 3.



What is the x-intercept (zero)? Answer: x = 0.

Practice (10 minutes approx.)	U-DO: An exercise in the manufacturing design of soup cans. Given the volume of a cylinder is $\pi r^2 h$; and that 237 cm^3 = 1 cup (approx); set the volume of your can design to be 1 cup or 237 c m^3 or: $237 = \pi r^2 h$; which has two variables, the radius, r of the can and the height, h, of the can. The job of the can designer is to choose the best dimensions. Ask the students what would be some of the factors that would determine a good choice of these variables in a soup can. Some factors would be store shelf height, home storage shelf height, visually appealing, and less material cost for largest volume. The last factor was very appealing to the manager of the project, so the equations for surface area were added to the solution: for a can, there is the top, bottom and cylinder: surface area = top + bottom + cylinder or surface area = $\pi r^2 + \pi r^2 + 2\pi rh$. Using the first criteria, the amount of soup to package in the can, solve it for 'h' and then substitute that value into this surface area equation. Then plot the resulting equation to find the minimum value of the surface area, which means least amount of aluminum material to make the can. Surface area = $\pi r^2 + \pi r^2 + 2\pi r(237/\pi r^2) = 2\pi r^2 + 474/r = (2\pi r^3 + 474)/r$ Minimum surface area is where r = 3.4 cm. The corresponding value of h is 6.5 cm.
Wrap-up (2 minutes approx.)	Wrap up closing comments and housekeeping.