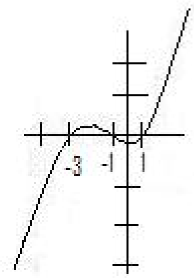
M^2 = Math Mediator Lesson 41: Polynomial Solutions

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises based on yesterday's lesson on polynomial zeros: 1. Find the zeros for: $4x^3 - 7x^2 - 21x + 18$ by graphing this polynomial on your graphing calculator. Answer: From the TABLE of the graph, students should find values of y = 0 for x values of -2, 3, and some number between 0 and 1. Show students method for finding the other zero on the graphing calculator: a. Go to CALC button from GRAPH b. Select ZERO from list c. Set 'Left Bound' to 0, press ENTER d. Set 'Right Bound' to 1, press ENTER e. Press ENTER for Guess?, or enter a guess value (i.e. 0.5) f. The answer of 0.75 should appear. (3/4 from yesterday).
Direct Instruction: (5 minutes approx.) **CA Std 3.0 and 10.0**	FUNDAMENTAL THEOREM OF ALGEBRA: Summarized by the statement, "for a polynomial of degree 'n', it will have 'n' zeroes." For the expression: $f(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$ how many zeroes does it have? Answer: Since the degree is 4, it will have 4 zeros. How many solutions for this expression: $0 = x^2 + 12x + 36$? Answer: The degree is 2, and there are two solutions, so to speak, but they are both the same since this is a square: $(x + 6)^2 = 0$ equivalent expression.
Practice: (3 minutes approx.)	U-DO: 1. Given: $f(x) = 6x^3 + 8x^2 + x - 6$; how many zeros will there be? Answer is 3, because the degree is 3. 2. Given: $0 = x^5 + 4x^4 + 2x^3 - 2x^2 - 4x + 1$ how many solutions will this have? Answer: Degree is 5, so 5 solutions.
Direct Instruction and Review: (5 minutes approx.)	From a few lessons back (Lesson # 36) it was shown how 3rd and 4th degree polynomials look like when graphed. If the polynomial is factored into neat binomial products with a monomial constant; such as the form of : $f(x) = a(x + 3)(x + 1)(x - 1)$ we can create an approximate graph easily.

<table>
<tr><td></td><td>f(x) = a(x + 3)(x + 1)(x – 1)
Odd order, leading coefficient positive.</td><td>
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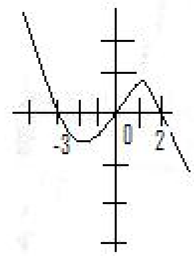
<tr><td></td><td>f(x) = -a(x + 3)(x)(x – 2)
Odd order, leading coefficient negative.</td><td>
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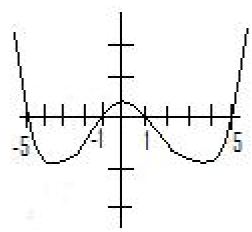
<tr><td></td><td>f(x) = a(x + 5)(x + 1)(x – 1)(x – 5)
Even order, leading coefficient positive.</td><td>
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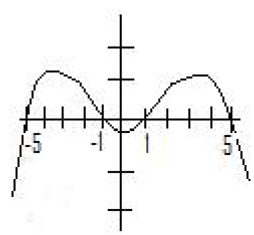
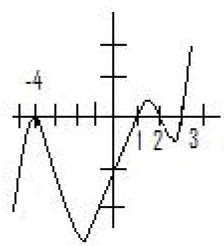
Odd order, leading coefficient negative.</td><td>
</td></tr>
<tr><td rowspan="2">Direct Instruction: (7 minutes approx.)</td><td>Take a look at the plot on the right and think about how many zeros and what degree the polynomial that describes it might be. Compare this plot with the 3rd and 4th order plots from above to see if there is a pattern that can be established.</td><td>
</td></tr>
<tr><td colspan="2">Since the start and finish are in opposite directions (up and down), it cannot be an even number, as the number of ‘x’ axis (or zeros) crossings might lead students to believe. Therefore, one zero must repeat, or else there is a complex number zero.</td></tr>
</table>

The next bit of information is to show how to create an equation for this graph. There are 4 zeros (3 crossings and one just touches), which are x = 1, 2 , 3 and -4 touches. Therefore, the equation can be started with this:

$$f(x) = a(x - 1)(x - 2)(x - 3)(x + 4)(x + ?)$$

First step in finding the missing factor is to assume that one of the zeros repeats itself, and typically, that zero is the one that just touches the x axis. So, going on that assumption, have the **<u>students graph</u>** that on their calculators, with a = 1 since the graph starts pointing downward and that matches with the 3^{rd} order starting downward; and see if it is close to the graph given. It should! But the y scale is in 100's.

Direct Instruction and Practice (15 minutes approx.)

Another way to create an equation from a set of data is by using the "Regression Feature" on your calculator. Have students input the following data about sport athletes into a list in their calculators.

Table 1: Athletic performance with age of athlete

Age	11	19	27	35	43	51	59	67	75
% Performance	40	85	98	92	88	80	75	65	40

Create a list: Step #1: Clear LISTS: Press "STAT" button and select 'ClrList', then press "LIST" button and select L1, press enter. This should have cleared list L1. Do the same with list L2.

Step #2: Enter Data: Press "STAT", select 'Edit', press enter. You should see three columns for lists L1, L2, and L3. Select the position of L1(1) and enter 11. Scroll down to L1(2) and enter 19. Repeat for the rest of the data, L2 is % performance.

Step #3: Display Data: Press "STAT PLOT", Select Plot1, press enter, select ON, select list L1 for x and L2 for y. Adjust WINDOW for 0 to 100; x and y axis. Clear any equations from "Y=" if there are other graphs displayed.

Step #4: Looking at the graph, decide if the plot is linear, quadratic, cubic, or quartic? Quartic is the best description. Press "STAT", select "CALC", select 7:QuartReg, press enter. Press "LIST" and select L1 to add L1 to the argument. Press ',' (comma) key, Press "LIST" and select L2. Press ENTER. Students should get an equation with variables defined.

Step #5: Write the equation given by the calculator:

$$(-5.7e^{-5})x^4 - 0.0107x^3 - 0.735x^2 + 21.07x - 116$$

Step #6: Enter the equation into the 'Y=' on the calculator.

Step #7: Press 'GRAPH' and observe how closely it matches the other graph. There are computer software tools that will do this also.

<table>
<tr><td>Practice
(5 minutes approx.)</td><td>U-DO:
1. Enter this set of data into the calculator, graph it, find an equation, and check the equation against the original graph.
<table>
<tr><td>L1(x)</td><td>0</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td></tr>
<tr><td>L2(y)</td><td>40</td><td>60</td><td>80</td><td>60</td><td>70</td><td>75</td><td>85</td><td>100</td></tr>
</table>
Answer: This graph better fits a cubic pattern, so the CubicReg is used for the equation: $0.012x^3 – 0.49x^2 + 6.7x + 41.2$ Which is close but not very accurate. If you do a Quartic Regression, it is a little bit closer in the range of data.

Note: The approximation may not always precisely fit, so depending on how accurate you need your equation to be, you would determine if you could use it or not. Computer programs are not limited to Quartic, so they will have better results.</td></tr>
<tr><td>Practice
(5 minutes approx.)</td><td>Using your calculator, write a function that best fits the following sets of data points. Use the format: f(x) = a(x ± b) (x ± c) (x ± d)etc…

1. (-1,0), (2,0), (0,3), (3,0) Answer: f(x) = a(x + 1)(x – 2)(x – 3) and to find ‘a’ they should input some numbers and tune it in by graphing them. The value of ‘a’ is 0.5.

2. (-5,0), (0,105), (3,0), (7,0) Answer: f(x) = 1(x + 5)(x – 3)(x – 7)</td></tr>
<tr><td>Wrap-up
(2 minutes approx.)</td><td>Wrap up closing comments and housekeeping.</td></tr>
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