M^2 = Math Mediator Lesson 40: Polynomial Zeros

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises based on yesterday's lesson on polynomial division: 1. Use long division to solve: $\frac{6x^3+8x^2+x-6}{3x-2}$ Answer: $2x^2 + 4x + 3$
Motivation Direct Instruction: (10 minutes approx.) **CA Std 3.0 and 10.0**	Polynomials are used in many careers as mathematical model for characteristics. Some examples are: ➢ Tourism prediction (Number of tourists vs. months of year) ➢ Guitar Design (Frequency vs. resonance gain) ➢ Economic models ➢ Medical models (*internet search "polynomial medicine model"). Some polynomials when graphed cross the 'x' axis and therefore have real solutions for y = 0. Some polynomials have complex solutions, as was shown in the last lesson on long division. The real points, where the graph of the polynomial crosses the 'x' axis are important points, called "zeros." If the polynomial f(x) = $6x^3 + 8x^2 + x - 6$ can be divided by 3x – 2 without producing a remainder, then 3x – 2 is a factor of the polynomial. From the Total Recall today, we found 2 factors of the expression $6x^3 + 8x^2 + x - 6$; namely (3x – 2) and $(2x^2 + 4x + 3)$. If the points where the graph crosses the 'x' axis needed to be found, then just set the expression to zero, and solve. In this case: f(x) = 0 = $6x^3 + 8x^2 + x - 6 = (3x - 2)(2x^2 + 4x + 3)$ and one solution would be 0 = 3x – 2 or x = 2/3. The point: x = 2/3 or (2/3, 0) is a ZERO of the function.
Direct Instruction and Practice: (10 minutes approx.)	U-DO: Given: f(x) = $4x^3 - 7x^2 - 21x + 18$ Try to find the zeros on your calculator. After a minute, students should find -2 and 3 and a point between 0 and 1. Another method to find the zeros of a polynomial is to use the relationship: $\frac{p}{q} = \frac{factor\_of\_the\_constant\_term}{factor\_of\_the\_coefficient\_of\_the\_leading\_term}$ From the given, the constant term is 18 and the leading term coefficient is 4. U_DO: One $\frac{p}{q}$ possible solution is $\pm\frac{18}{4}$, find all the others. Possible solutions are: $\pm\frac{18}{4}, \pm\frac{9}{4}, \pm\frac{3}{4}, \pm\frac{1}{4},$ $\pm\frac{9}{1}, \pm\frac{9}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{1}{2}, \pm 1, \pm\frac{18}{1}, \pm\frac{6}{1}, \pm\frac{2}{1}$ Which ones are between 0 and 1? $\frac{3}{4}, \frac{1}{4}, \frac{1}{2}$

Direct Instruction: (5 minutes approx.)

Using SYNTHETIC DIVISION to find the correct zeros. We have used LONG DIVISION, now we will show how Synthetic Division works.

Demonstration: It was already found that -2 was one solution, from the graphing calculator. In order to use synthetic division, write down all the coefficients from the expression, and then put the number to divide by next to it: The variables and the order of the variable is placed above the coefficients to show their position, but this is not used in the actual synthetic division.

$$\begin{array}{r} x^3 \quad x^2 \quad x \quad const \\ -2\overline{)\,4 \quad -7 \quad -21 \quad 18} \end{array}$$

Step #1: $-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$

Drop down the first number: **4**

Step #2:

Multiply it by the divisor and place that product $-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$

under the second number (-2 x 4 = -8) 4 **-8**

Step #3:

$-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$

Add that product to the second 4 -8

And place the result under the product **-15**

Step #4:

$-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$

Multiply that addition result by the divisor and 4 -8 **30**

Place the result under the third number: -15

Step #5:

$-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$

Add that product to the third number 4 -8 30

Place the result under the product: -15 **9**

	Step #6: $-2\overline{)\,4 \quad -7 \quad -21 \quad 18}$ Multiply that addition result by the divisor and 4 -8 30 **-18** Place the result under the third number: -15 9 **Step #7:** Adding the final numbers: 18 + (-18) = 0, thus there is NO remainder. NO REMAINDER = FACTOR
Practice (10 minutes approx.)	U-DO: 1. Perform synthetic division with 3 on the same expression. 2. Perform synthetic division with ¾ on the same expression. Solution: $\frac{3}{4}\overline{)\,4 \quad -7 \quad -21 \quad 18}$ 4 3 -3 **-18** -4 -24 NO REMAINDER!
Practice (10 minutes approx.)	U-DO: Using the same technique, find the zeros for $8x^3 - 35x^2 + 36x - 9$ 1. Use the calculator to see if zeros are easy, whole numbers and if not, find out the numbers the other zeros will be between. 2. List all the possible factors using p/q . 3. Verify all the zeros with synthetic division: 1, 3 and 3/8.
Practice (5 minutes approx.)	U-DO: 1. Polynomials can be used to solve geometric problems. In order to make a pyramid pudding mold that is 0.5 feet higher that it is wide, we can use the expression Volume = 1/3(bh) where h = 0.5 + b and the base $b = x \cdot x = x^2$. Graph and solve this on your calculator if the volume of pudding for all the people attending needs to be 3 cubic feet. Answer: x = 2 feet.
Wrap-up (2 minutes approx.)	Wrap up closing comments and housekeeping.