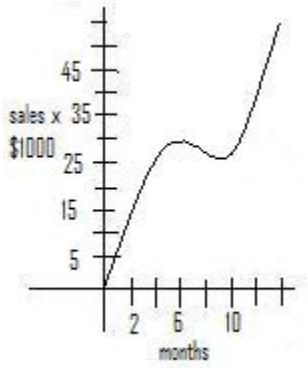
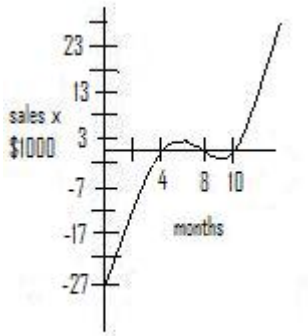
M^2 = Math Mediator Lesson 38: Polynomial Factoring

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises based on yesterday's lesson on polynomial operations: 1. Perform the following polynomial operation and simplify: $(3x^3 + 4x^2 + 2x + 7) – (4x^3 + 2x^2 – 3x + 5) = ?$ Answer: $-x^3 + 2x^2 + 5x + 2$ 2. $(5x^4 + 3x^2 + x – 5) + (3x^4 – 2x^3 + 3x^2 – 4x + 8) = ?$ a. Answer: $8x^4 – 2x^3 + 6x^2 – 3x + 3$ 3. $(x – 2)^3 = ?$ Answer: $x^3 – 6x^2 + 12x – 8$
Direct Instruction: (10 minutes approx.) **CA Std 3.0, 4.0 and 10.0**	MOTIVATION: Many career fields use polynomials to model trends of data. For instance, the health and insurance industry use polynomials to track and predict disease epidemic spreading and the probability of how many people will be infected. Another industry that uses polynomials is the business of fashion clothing. For example, a new style of clothing might be developed and released in Italy, where it gains popularity over a few months, but then sales slow down until the style gains acceptance in another country (United States), when the sales take off again. This could be plotted with the following graph:   then shifting the x-axis up: Now, there are some zeros from which we can create an equation or function representing the sales trend of the new fashion design. $f(x) = a(x - 4)(x - 8)(x - 10)$ and then solve for 'a' by using the point (0, -27) $-27 = a(0 – 4)(0 – 8)(0 – 10) = -a320$ and $a = 27/320$ however, this function must be offset by the -27,000 in order to give us accurate data. Therefore, we can just add 27 to the function to be precise. $f(x) = \{27/320(x – 4)(x – 8)(x – 10)\} + 27$ Then, we would plot this function and compare it to the data and see if it is a good fit. If it is, we have an equation that describes the sales trend. Using the original solution, the clothing manufacturer might wish to divide by the total number of stores caring the fashion to track the average sales per store. If the data showed that the number of stores followed the trend of: $f(x) = (x – 4)(x – 8)(x – 13)$; then dividing the polynomials would give the sales per store. Sales per store (average): $\frac{\frac{27}{320}(x-4)(x-8)(x-10)}{(x-4)(x-8)(x-13)} = \frac{\frac{27}{320}(x-10)}{(x-13)}$ where 'x' is in

	months.
Direct Instruction: (5 minutes approx.)	Factoring Polynomials (Tips to factoring): 1. Look for GCF (Greatest Common Factor) when simplifying; a. $3x^2 + 9x = 3x(x + 9)$ 2. Look for trinomial patterns: a. Perfect Squares: $x^2 + 10x + 25 = (x + 5)^2$ b. Difference of squares (no middle term): $4x^2 – 1 = (2x + 1)(2x – 1)$ c. Trial and error to factor out a binomial: $x^2 + 4x + 3$ (Since the sign on the last term is positive, you know that the binomial factors must both be the same operation, addition or subtraction. Since the sign on the middle term is positive, then you know that both the signs are positive): $(x + ?)(x + ?)$ The next step is to find two numbers that when multiplied give you the last number 3, but when added give you the coefficient to the middle term, 4. Those numbers are 3 and 1: $(x + 3)(x + 1) = x^2 + 4x + 3$
Practice: (10 minutes)	U_DO: Factor the following polynomials: 1. $8x^2 + 40x$ Answer: $8x(x + 5)$ 2. $x^2 + 12x + 36$ Answer: $(x + 6)^2$ 3. $x^2 – 16$ Answer: $(x + 4)(x – 4)$ 4. $z^3 + 2z^2 – 15z$ Answer: $z(x + 5)(x – 3)$
Direct Instruction: (10 minutes approx.)	More Factoring Polynomial Tips: 3. Difference of two cubes: $27 - z^3 = 3^3 - z^3 = (3 – z)(9 + 3z + z^2)$ a. General Form: $a^3 - b^3 = (a – b)(a^2 + ab + b^2)$ 4. Sum of two cubes: $x^3 + 64 = x^3 – 4^3 = (x + 4)(x^2 – 4x + 16)$ a. General Form: $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
Practice (5 minutes approx.)	U-DO: Factor the following polynomials: 1. $t^4 – 8t$ Answer: $t(t^3 – 8) = t(t – 2)(t^2 + 2t + 4)$ 2. $4x^4 + 108x$ Answer: $4x(x^3 + 27) = 4x(x + 3)(x^2 – 3x + 9)$
Direct Instruction: (5 minutes approx.)	More Factoring Polynomial Tips: 5. Factor by common grouping: $r^3 – 2r^2 – 9r + 18 = r^2(r – 2) – 9(r – 2) = (r^2 – 9)(r – 2) = (r + 3)(r – 3)(r – 2)$ 6. Using factoring to solve polynomial equations: a. $2y^3 - 14y^2 = -24y$ Solve: $2y^3 - 14y^2 + 24y = 0$ $2y(y^2 - 7y + 12) = 0$ then $2y(y – 3)(y – 4) = 0$ and $y = 0, 3, 4$
Practice	U-DO: Factor the following polynomial function that forecasts a trend in sales:

(3 minutes approx.)	1. $3m^3 - 33m^2 + 72 = 0$ where m is in months. Answer: $3m(m - 3)(m - 8) = 0$
Wrap-up (2 minutes approx.)	Wrap up closing comments and housekeeping.