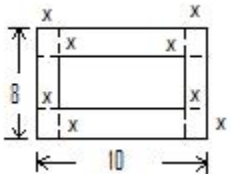
M^2 = Math Mediator Lesson 36: Polynomial Functions

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises based on the definition of *polynomial*: **A *polynomial* is a monomial or any sum of monomials.** 1. Which of the following are monomials, which are polynomials? a. 7 (Answer: 7 is both a monomial and polynomial, it is also a constant). b. $3x^2 + 2x + 3$ (Answer: it is a polynomial, sum of monomials) c. 2xy (Answer: Both monomial and polynomial. There are three terms in this polynomial, 2 is constant, x and y are variable terms.). d. $3t^2v$ (Answer: Both monomial and polynomial.)
Direct Instruction and Activity: (20 minutes approx.) **CA Std 3.0, 4.0 and 10.0**	Previous lessons involved quadratics: $y = ax^2 + bx + c$ (Quadratic means that there is a term taken to the 2nd degree, here is shown in standard form). We presented a use of quadratics was for projectiles or dropped objects, how gravity effected those objects. Quadratics produce parabolas when graphed. Parabolas are used in graphic design (logos) and lampshade design. A quadratic function is expressed as $f(x) = ax^2 + bx + c$. The area of a rectangle can produce a quadratic, given the length and width of the sides as '2x' and 'x', and knowing the area of a rectangle is length times width or $A = lw = 2x(x) = 2x^2$. Adding another dimension to the area of a rectangle gives us volume. Volume of a rectangle is length times width times height or V = l x w x h. Today, we will construct several 'pans' from 8" x 10" construction paper. We will use different heights to see how the volume changes and then plot the solutions. Start by handing out at least 6 pieces of construction paper. Have students use the template below to construct the pan from the paper, and then record the data in a table, shown:  1. Make several 'pans' from construction paper: a. Cut squares of 'x' dimension from corners. b. Fold sides up and tape corners. c. Measure length, width and height, calculate volume. d. Enter volume into the table. 2. Write a function that describes the volume of the 'pans', in terms of 'x'. a. Answer: $f(x) = (10 - 2x)(8 - x)x$ 3. Graph the function on your calculator. What are the y = 0 points (hint:

x	volume
1"	
1.5"	
2"	
2.5"	
3"	
3.5"	

use x = 0, 4 and 5)? What is the maximum value of y? What is the maximum volume?

4. Use the standard form for polynomials: $f(x) = (x - r_1)(x - r_2)(x - r_3)$ with $r_1 = 0$; $r_2 = 4$; and $r_3 = 5$ (zeros or roots from #3). Plot this equation on your calculator: $f(x) = (x - 0)(x - 4)(x - 5)$. Does it look the same as the previous plot from #3? No, because we forgot to multiply by the 'a' value of 4, which we factored out of the original function. $f(x) = a(x - 0)(x - 4)(x - 5) = f(x) = 4(x - 0)(x - 4)(x - 5)$.

Terms: (10 minutes approx.)

POLYNOMIAL TERMS:

Degree	Type	Example
0	Constant	$f(x) = 3$
1	Linear	$f(x) = 2x + 3$
2	Quadratic	$f(x) = 2x^2 - 12x + 72$
3	Cubic	$f(x) = 4x^3 - 36x^2 + 80x + 0$
4	Quartic	$f(x) = x^4 + 2x^3 + 3x^2 - 2x - 6$

**Polynomials are NOT expressions with variables as exponents: 2^x or expressions with the variable raised to a non-whole number, such as 1.5 or -1, where the variable is in the denominator: $1/(2x)$.

U-DO: Which of the following are polynomials?

1. $16x^2 - 36x - 2^x$ Answer: Not a polynomial, variable is exponent.
2. $8x^3 + 24x^2 - 36x^{1.5} + 36x - 72$ Answer: Not a polynomial, exponent is not whole number.
3. $8x^2 - 36x - 72$ Answer: Yes, it is a polynomial. Degree 2, quadratic.
4. $4x^2 + 16x - \frac{4}{x} + 2$ Answer: No, variable is in denominator.

Practice: (5 minutes)

U-DO: Find f(x) for x = 2; where $f(x) = x^4 + 2x^3 + 3x^2 - 2x - 6$

Answer: $2^4 + 2(2^3) + 3(2^2) - 2(2) - 6 = 16 + 16 + 12 - 4 - 6 = 34$

Practice (10 minutes approx.)

U-DO: Plot the following equations on your graphing calculator. Using the zeros from the plot, re-write the equations in root form: $f(x) = (x - r_1)(x - r_2)(x - r_3)$

1. $f(x) = 4x^3 + 8x^2 - 36x - 72$ Answer: $f(x) = 4(x - 3)(x + 2)(x + 3)$
2. $f(x) = 0.5(x^4 + x^3 - 42x^2 - 36x + 216)$ Answer: $0.5(x + 6)(x + 3)(x - 2)(x - 6)$
3. $f(x) = 3x^3 + 15x^2 - 12x - 60$ Answer: $3(x + 5)(x + 2)(x - 2)$

Wrap-up (5 minutes approx.)

Wrap up closing comments and housekeeping.

