# $M^2$ = Math Mediator Lesson 35: Exponents

!! THIS LESSON REQUIRES AT LEAST ONE DICE, PREFERABLY SEVERAL, ONE FOR EACH STUDENT!!

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| Intro: (3 minutes approx.) | Today's lesson is on using exponents: Some areas where exponents are used are:<br>Chemists: Subatomic particles have a mass of $1.67 \times 10^{-24}$ grams (proton).<br>Astronomers: Some stars are $2.5 \times 10^{13}$ miles away from Earth (Alpha Centari).<br>MP3 players: How many songs can a 120 GB MP3 hold, each song is 4 MB?<br>$1.2 \times 10^{10}$ Bytes ÷ $4 \times 10^{6}$ Bytes per song = 3000 (Songs)<br>We encounter and need exponents in many areas of life. |
| Activity: (15 minutes approx.) ***Dice Needed!***<br>**CA Std 12.0** | 6 Year Fungus: Each student stands and throws a die (6 sided). Whoever throws a 6 sits down. Record the number of students standing after each throw of the dice (i.e. 0 = 25; 1 = 18; 2 = 9; 3 = 5, …).<br>Plot the data. Where does the 'half life' occur (where half of the students are standing)?<br>Plot the function: y = (total number of students) $\cdot 0.818^x$ (where 'x' is the number of the dice throw) and compare to the other plot. |
| Direct Instruction and Practice: (10 minutes approx.) | Exponential functions: $y = ab^x$ (general form)<br>Two cases of an exponential function are a) growth; or b) decay.<br>If $b < 1$ : the exponential function is a decay function with a "half life."<br>If $b > 1$ : the exponential function is a growth function with a "doubling" time.<br>U-DO: If the growth rate of the population of India is expressed as the following exponential function: y = (current population) $\cdot 1.02^x$ ; where 'y' is the population for 'x' years and the current population is 1.2 billion:<br>How many years will the population double? Answer: The solution is the value of 'x' in years where $1.02^x = 2$; students can plug in values in their calculator and find that x is close to 35 years. |
| Direct Instruction: (3 minutes approx.) | Properties of Exponents: Multiplication of exponents.<br>$2^2 \cdot 2^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 2^5$<br>$3^3 \cdot 3^5 = 3^8$ as well as negative exponents: $4^3 \cdot 4^{-2} = 4^1$<br>Multiplying exponents with the same base number, just add the exponents.<br>Generally speaking: $a^m \cdot a^n = a^{m+n}$ |
| Direct Instruction: (3 minutes approx.) | Properties of Exponents: Division of exponents.<br>$\frac{2^4}{2^2} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2 \cdot 2 = 2^2$<br>$\frac{5^2}{5^3} = \frac{5 \cdot 5}{5 \cdot 5 \cdot 5} = \frac{1}{5} = 5^{-1}$ |

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| | $\frac{6^5}{6^{10}} = \frac{6\cdot6\cdot6\cdot6\cdot6}{6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6} = \frac{1}{6\cdot6\cdot6\cdot6\cdot6} = 6^{-5}$<br>Dividing exponents with the same base number: subtract exponents.<br>General form: $\frac{a^m}{a^n} = a^{m-n}$ |
| Direct Instruction: (3 minutes approx.) | Properties of Exponents: Exponent to an exponent.<br>$(2^3)^2 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2) = (2\cdot2\cdot2\cdot2\cdot2\cdot2) = 2^6$<br>$(5^4)^3 = (5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)\cdot(5\cdot5\cdot5\cdot5)$<br>$= (5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5) = 5^{12}$<br>Taking an exponent to another exponent, or a power to a power, multiply exponents.<br>General form: $(a^m)^n = (a^{mn})$ |
| Practice: (3 minutes approx.) | Using multiplication or division of exponents, show that $a^0 = 1$<br>Answer: $3^3 \cdot 3^{-3} = 3^0 = \frac{3\cdot3\cdot3}{3\cdot3\cdot3} = 1$ or $\frac{2^2}{2^2} = 2^0 = 1$ |
| Direct Instruction: (2 minutes approx.) | $(4\cdot3)^3 = (4\cdot3)\cdot(4\cdot3)\cdot(4\cdot3) = 4\cdot4\cdot4\cdot3\cdot3\cdot3 = 4^3\cdot3^3$<br>General form: $(a\cdot b)^m = (a^m\cdot b^m)$ |
| Direct Instruction: (2 minutes approx.) | For division: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ Remember: $a^{-n} = \frac{1}{a}$ so: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$<br>If a = b; then $a^m = b^m$ as well as; (Common power property)<br>If $a^m = a^n$ then m = n. (Common base property) |
| Practice: (10 minutes approx.) | U-DO: 1. $9^x = 27$ solve for 'x': Answer: $(3^2)^x = 3^3$ and using the common base property 2x = 3 and therefore x = 3/2.<br>2. $16^x = (1/32)$ solve for 'x': Answer: $(2^4)^x = 2^{-5}$ and then 4x = -5; x = -5/4<br>3. $\left(\frac{64}{49}\right)^x = \left(\frac{7}{8}\right)^{\frac{2}{3}}$ solve for 'x': $\left(\frac{8^2}{7^2}\right)^x = \left(\frac{8}{7}\right)^{\frac{-2}{3}}$ and then 2x = -2/3; x = -1/3 |
| Wrap-up (2 minutes approx.) | Wrap up closing comments and housekeeping. |