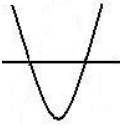
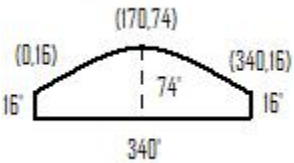
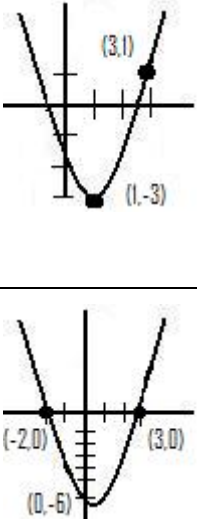


# M<sup>2</sup> = Math Mediator Lesson 34: Parabolic Graphs

<p>Total Recall (Warm-up) (10 minutes approx.)</p>	<p>Total Recall: 2 exercises from yesterday's lesson on the Quadratic Formula.</p> <p>1. Which discriminant would describe this parabola that crosses the x-axis?</p> <p>a. <math>\sqrt{42^2 - 4(7)(60)}</math>      b. <math>\sqrt{2^2 - 4(2)(5)}</math></p> <p>c. <math>\sqrt{12^2 - 4(2)(18)}</math>      d. <math>\sqrt{9^2 - 4(1)(30)}</math></p> <p>Answer: If the discriminant is greater than zero, there are two real roots for the solution of a quadratic equals zero. Therefore, the only solution that produces an answer greater than zero is 'a'.</p> <p>2. Using the solution discriminant of the above exercise, and the formulas for quadratics in vertex and standard form, find the point of the vertex, the quadrant that it is in, and if it is a max or min?</p> <p><math>y = ax^2 + bx + c</math> (standard form)</p> <p><math>y = a(x - h)^2 + k</math> (vertex form)</p> <p>(carry out the square: <math>y = ax^2 - a2xh + ah^2 + k</math>)</p> <p>Therefore: <math>y = ax^2 + bx + c = a(x - h)^2 + k = ax^2 - a2xh + ah^2 + k</math> and comparing terms: <math>ax^2 = ax^2</math> and <math>bx = a2xh</math> (or <math>b = a2h</math>) and finally <math>c = ah^2 + k</math>. Furthermore, solving for h, <math>h = b/(2a)</math> and solving for k: <math>k = c - b^2/(4a)</math>.</p> <p>Answer: <math>h = 42/((2)(7)) = 3</math>; <math>k = 60 - 42^2/((4)(7)) = -3</math>. Therefore, the vertex is at (3,-3) which is in the 4<sup>th</sup> quadrant and it is minimum.</p>	
<p>Direct Instruction (10 minutes approx.)</p> <p>CA Std 8.0</p>	<p>Parabolic arches are used in the construction of bridges and buildings. An aircraft hanger building in Mine uses a parabolic arch shaped roof. Today, we will discover when given the following dimensions, how to find the quadratic equation describing the shape of the roof.</p>  <p>1. To solve this, use vertex form (<math>y = a(x - h)^2 + k</math>) and describe the vertex point with 'h' and 'k'.</p> <p><math>y = a(x - 170)^2 + 74</math> (h = 170 and k = 74)</p> <p>Next, use the point assigned to where the roof meets the wall (0,16) as one of the solution points (x,y) given and plug it into the equation and solve for 'a'.</p> <p><math>16 = a(0 - 170)^2 + 74</math> ; solving for a: <math>a = -0.002</math></p> <p>The equation in vertex form is then: <math>y = -0.002(x - 170)^2 + 74</math></p> <p>2. Another method is to use the quadratic intercept form: <math>y = a(x - p)(x - q)</math> ; where p and q are x-intercepts (where the parabola crosses the x-axis).</p>	

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	<p>a. Imagine the x-axis is at the points where the roof meets the side walls. The points are given as (0,16) and (340,16), but with the x-axis shifted up to the top of the walls, the points would be (0,0) and (340,0). Therefore, <math>p = 0</math> and <math>q = 340</math>. Substitute these into the intercept form: <math>y = a(x - 0)(x - 340)</math>. Next we need to select a point on the parabola and substitute it into the equation to find 'a'. We could use (0,0), however that gives us 'a' times 0 which is zero and we cannot divide y by zero. Same with (340,0), because <math>(340 - 340)</math> equals zero and then we would again divide by zero. The vertex can easily be found from the point (170,74), but remember that we shifted the x-axis up 16 feet, so the actual point is <math>(170, (74-16)) = (170, 58)</math></p> <p><math>58 = a(170 - 0)(170 - 340)</math> and solving for a: <math>a = -0.002</math>.</p> <p><b><math>y = -0.002(x)(x - 340)</math></b></p> <p>**Exercise: plot both on calculator, notice how the shape is the same, just shifted up by the 16 feet.</p>	
<p>Practice: (10 minutes approx.)</p>	<p>U-DO: 1. Write in vertex form the quadratic equation for the parabola shown to the right.</p> <p>Answer: <math>y = a(x - h)^2 + k</math> ; (<math>h = 1</math>; <math>k = -3</math>)  <math>y = a(x - 1)^2 - 3</math> (use point (3,1) and solve for 'a')  <math>1 = a(3 - 1)^2 - 3</math> ; <math>a = 1</math>          Vertex form: <math>y = (x - 1)^2 - 3</math> (Note: when you use the vertex point to solve for 'a', you get <math>0 = 0</math>)</p> <p>2. Write in intercept form the equation for the parabola shown to the right.</p> <p>Answer: <math>y = a(x - p)(x - q)</math> ; (<math>p, q</math> are x-intercept)  <math>y = a(x - 3)(x - (-2))</math> ; (use point (0,-6), solve 'a')  <math>-6 = a(0 - 3)(0 + 2)</math> ; <math>a = 1</math>          Intercept Form: <math>y = (x - 3)(x + 2)</math></p>	
<p>Direct Instruction: (10 minutes)</p>	<p>Another form is STANDARD form: <math>y = ax^2 + bx + c</math> ; and when <math>y = 0</math> we learned in an earlier lesson that the values of a, b and c are used in the Quadratic Formula to solve for 'x'. If a, b and c are not given, but 3 points on the parabola are known, we can create 3 equations and solve as we did for systems of equations.</p> <p>Example: Three points on a parabola are: (3,0); (5,30); and (-1,-12). The three equations are (using standard form and substituting for 'x' and 'y'):</p> <ol style="list-style-type: none"> <li>1. <math>0 = a(3^2) + b(3) + c</math> or <math>0 = 9a + 3b + c</math></li> <li>2. <math>30 = a(5^2) + b(5) + c</math> or <math>30 = 25a + 5b + c</math></li> <li>3. <math>-12 = a(-1^2) + b(-1) + c</math> or <math>-12 = a - b + c</math></li> </ol> <p>Using the elimination method to solve systems of equations: first take equations 1 and 2 and combine to remove a variable ('c'). Then take equations 2 and 3 to combine and remove the same variable. Then take the resulting two variable equations and combine to remove a variable, solving for one variable.</p>	

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	<p>1 &amp; 2: subtract Eq.1 from Eq.2: <math>30 - 0 = (25a - 9a) + (5b - 3b) + (c - c)</math> ; which results in: <math>30 = 16a + 2b</math> ; simplify by dividing by 2: <math>15 = 8a + b</math>.</p> <p>2 &amp; 3: subtract Eq.3 from Eq.2: <math>30 - (-12) = (25a - a) + (5b - (-b)) + (c - c)</math>; Which results in: <math>42 = 24a + 6b</math> ; simplify by dividing by 6: <math>7 = 4a + b</math>.</p> <p>Now subtract the resulting equations: <math>15 - 7 = (8a - 4a) + (b - b)</math> ; which results in: <math>8 = 4a</math> and then: <math>a = 2</math> .</p> <p>Substitute <math>a = 2</math> into one of the resulting equations: <math>7 = 4(2) + b</math>; <math>b = -1</math>.</p> <p>Substitute <math>a = 2</math> and <math>b = -1</math> into one of the first 3 equations: <math>0 = 9(2) + 3(-1) + c</math>; and we find that <math>c = -15</math>.</p> <p>Putting these values into the Standard form: <b><math>y = 2x^2 - x - 15</math></b> is the solution!</p>
<p>Practice (10 minutes approx.)</p>	<p>U-DO: Try this three point exercise:</p> <p>1. Given the following three points that are on a parabola, find the equation of the parabola in standard form: (1, 18); (-2, 0); and (-3, -10)</p> <p>Answer: <b><math>y = -x^2 + 5x + 14</math></b></p> <p>1. <math>18 = a(1^2) + b(1) + c</math> or <math>18 = a + b + c</math></p> <p>2. <math>0 = a(-2^2) + b(-2) + c</math> or <math>0 = 4a - 2b + c</math></p> <p>3. <math>-10 = a(-3^2) + b(-3) + c</math> or <math>-10 = 9a - 3b + c</math></p> <p>1 &amp; 2: <math>18 = -3a + 3b</math> or <math>6 = -a + b</math></p> <p>2 &amp; 3: <math>10 = -5a + b</math></p> <p>(1&amp;2) - (2&amp;3): <math>-4 = 4a</math> or <math>a = -1</math>; <math>10 = -5(-1) + b</math> or <math>b = 5</math>; And finally: Eq.1: <math>18 = -1 + 5 + c</math> or <math>c = 14</math>.</p>
<p>Wrap-up (5 minutes approx.)</p>	<p>Wrap up closing comments and housekeeping.</p>