M^2 = Math Mediator Lesson 33: Quadratic Formula

Total Recall (Warm-up) (5 minutes approx.)

Total Recall: 2 exercises from yesterday's lesson on Factoring Quadratics and Completing the Square:

1. Complete the square in this equation and put the resulting equation into vertex form: $x^2 + 12x + 30 = y$: A. $x^2 + 12x + 30 + 6 - 6 = y$; $x^2 + 12x + 36 - 6 = y$; $(x + 6)^2 - 6 = y$ (vertex form).

2. Solve for x in this quadratic: $2x^2 - 12x + 10 = 0$: A. Divide both sides by 2 to get $x^2 - 6x + 5 = 0$; factor to $(x - 5)(x - 1) = 0$; x = 5 or 1.

Direct Instruction (10 minutes approx.)

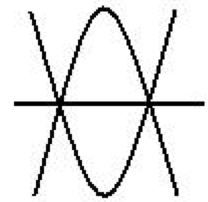
CA Std 8.0

Situation: Some people are starting up a new business, called MinMax Apparel. The slogan is "Minimum care/Maximum Comfort". They have quickly sketched a company logo, shown at the right. The logo consists of two identical parabolas, symmetrical and inverted. You need to create the logo precisely, in a computer controlled printer graphic, and using a formula for the parabola is one way. If the first parabola has a quadratic equation that defines the points as: $y = 7x^2 - 42x + 60$; how can the other parabola quickly be found (Hint: use the vertex form)? Remember the vertex form is:

$y = a(x - h)^2 + k$; so a will need to be 7 and from there we need to complete the square:

$y = 7(x^2 - 6x + 60/7)$; the nearest factor of 7 to 60 is 63 (7 x 9) so we will add and subtract 3 to the right side of the equation to complete the square: $y = 7x^2 - 42x + 60 + 3 - 3$; which is equal to: $y = 7x^2 - 42x + 63 - 3$; and further simplified to: $y = 7(x^2 - 6x + 9) - 3$ or finally in vertex form: $y = 7(x - 3)^2 - 3$

New Company Logo:



Answer for using two parabola quadratic equations to precisely identify the logo:

Parabola 1: $y = 7(x - 3)^2 - 3$

With vertex form, we learned that a positive coefficient on the square term determined that the parabola opened up. For the other parabola, we just need a negative coefficient and we need to move the vertex up 3 from the x-axis to be symmetrical.

Parabola 2: $y = -7(x - 3)^2 + 3$

The next bit of information that the graphics program will need is <u>where the parabolas in the logo intersect each other and cross the x-axis</u>. How do you suppose we can determine this? Yes! Set y to zero and solve for the x values:

$y = 0 = 7x^2 - 42x + 60$

(hard to factor, so we use a <u>new tool</u> called the <u>**Quadratic Formula:**</u>

where $0 = ax^2 + bx + c$; $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

using: $0 = 7x^2 - 42x + 60$ $x = \dfrac{-(-42) \pm \sqrt{(-42)^2 - 4(7)(60)}}{2(7)}$

	$x = \frac{-(-42) \pm \sqrt{1764 - 1680}}{14}$ $x = \frac{42 \pm \sqrt{84}}{14}$ $x = \frac{42 \pm \sqrt{(4)(21)}}{14}$ $x = \frac{42 \pm 2\sqrt{21}}{14}$ $x = \frac{21 \pm \sqrt{21}}{7} = \frac{21}{7} \pm \frac{\sqrt{21}}{7} = 3 \pm \frac{\sqrt{21}}{7} = 3 \pm 0.655$ Therefore, the x-intercepts are 3.655 and 2.345
Direct Instruction (5 minutes approx.) Terms:	Consider the following two equations from the logo: 1. $y = 7(x - 3)^2 - 3$ 2. $y = -7(x - 3)^2 + 3$ Which of them has a maximum y value? (#2 because the vertex is the maximum that y-values can be, and all other y-values are less) Which has a minimum y-value? (#1 because the vertex is the minimum y-value for all the values of y, and all other solutions for y are larger). Which opens up? (#1) Which opens down? (#2) Now, compare the equation #1 ($y = 7(x - 3)^2 - 3$); with $y = (x - 3)^2 - 3$. Do they share the same vertex? (yes) Which is "thinner" than the other? (Equation #1 has a 7 multiplier (value of 'a' in vertex form), so the y values will increase more and be 'thinner'.
Practice: (10 minutes)	U-DO: 1. $3x^2 = 5x + 8$ Find x. Answer: x = 8/3 and -1. 2. Dianne hits a softball straight up from 1 meter above ground at 30 meters/second. Find when it will touch the ground, using the time/height formula: $h = -5t^2 + v_0 t + h_0$ (gravity is divided by 2 and rounded to -5m/s). v_0 = 30 m/s and h_0 = 1 meter (given) : $h = 0 = -5t^2 + 30t + 1$ (h = zero because we want to know at what time the ball will hit the ground, height zero). Use the Quadratic Formula. Answer: $x = \frac{-30 \pm \sqrt{30^2 - 4(-5)(1)}}{2(-5)} = 3 \pm 3.03$ (Do both answers make sense? No, only the positive 6.03.) Extra Points: what is the maximum height? Answer: From the vertex form of $y = a(x-h)^2 + k$ and the quadratic $y = ax^2 + bx + c$; expand the vertex form to $ax^2 - 2axh + ah^2 + k$ kand comparing the two forms we get $b = -2ah$ and $c = ah^2 + k$; solving for $h = (-b/2a)$ and $k = c - (b^2/4a)$ and knowing that k is the vertex maximum; $k = 1 - (30^2/(4(-5))$ or $k = 1 - (900/-20)$ and height is 46 meters.
Practice and Instruction: (8 minutes approx.)	U-DO: 1. $0 = 2x^2 + 2x + 5$ Find x. Answer: Using the quadratic equation $x = \frac{-2 \pm \sqrt{2^2 - 4(2)(5)}}{2(2)} = \frac{-2 \pm \sqrt{-36}}{4}$ which indicates no solution, because there are no real numbers that can produce a square root of -36. Graph this on your calculator to prove it and see what the solution set is for $y = 2x^2 + 2x + 5$.

	2. -8 = 2x² + 12x + 10 Find x. Answer: $x = \frac{-12 \pm \sqrt{12^2 - 4(2)(18)}}{2(2)} = \frac{-12 \pm 0}{4}$ This answer only has one solution. How can this be? Graph it on your calculator and see why. It only intersects the x axis at the vertex, one point, not two. DISCRIMINANT: The term for $b^2 - 4ac$ which is what is under the radical. There are three possibilities with the Discriminant: 1. It is > 0 (This means there are two real solutions). 2. It is < 0 (This means that there are no real solutions) 3. It is = 0 (This means that there is one solution).
Practice (5 minutes approx.)	U-DO: Using the discriminant rule, describe the solution of zeros for the following equations: 1. $y = 3x^2 - 18x + 27$ Answer: 1 solution 2. $y = x^2 + 9x - 3$ Answer: 2 solutions 3. $y = x^2 + 9x + 30$ Answer: no real solution
Instruction and Practice (10 minutes approx.)	U-DO: A ride shoots you up to 400 feet maximum and then back down. We wish to find the initial velocity in ft/sec using the formula: $h = -16t^2 + v_0t + h_0$. Like before, we substitute what is given: 400 for h and 0 for h_0. But this time, v_0 is not given, and we are asked to find it. What do you know about maximum points and vertex form that will help to solve this problem? Answer: The maximum value of the vertex is found with vertex form (value of k) $y = a(x-h)^2 + k$ and the quadratic $y = ax^2 + bx + c$; expand the vertex form to $ax^2 – 2axh + ah^2 + k$ kand comparing the two forms we get $b = -2ah$ and $c = ah^2 + k$; solving for $h = (-b/2a)$ and $k = c – (b^2/4a)$ and knowing that k is the vertex maximum and: $k = c – (b^2/4a)$. We know that k is 400 feet, and that a = -16 and c is 0. We solve for b, which is v_0 and find that $400 = 0 – (b^2/(4(-16))$ or $b = \sqrt{400(64)} = 160$ feet per second.
Wrap-up (3 minutes approx.)	Wrap up closing comments and housekeeping.