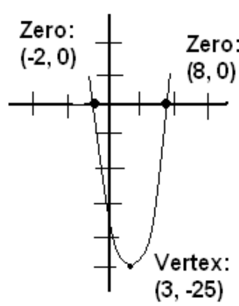
M^2 = Math Mediator   Lesson 32: Complete the Square

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises from yesterday's lesson 1. Add these complex numbers: (1 – 5i) + (-2 + 3i) Answer: (-1 –2i) 2. Multiply these complex numbers: (1 - 5i)(-2 + 3i) Answer: (13 + 13i) 3. Simplify: $\frac{1-5i}{-2+3i}$ Answer: $\frac{1-5i}{-2+3i}\cdot\frac{-2-3i}{-2-3i}=\frac{-2-3i+10i+15i^2}{4-9i^2}=\frac{-17+7i}{13}$
Direct Instruction (10 minutes approx.) **CA Std 6.0**	A couple lessons ago, it was shown that quadratics could be solved by finding the square root: $3r^2 = 48$; $r^2 = 16$; $r = \pm\sqrt{16} = \pm 4$. Last lesson showed that there is a representation for the square root of a negative number, and it is complex or imaginary. $3r^2 = -48$; $r^2 = -16$; $r = \pm\sqrt{-16} = \pm 4i$. We also discussed how quadratics can be shown in standard form: $ax^2 + bx + c$ or in vertex form: $a(x - h)^2 + k$, where the vertex is positioned at (h, k). This information, along with factoring quadratics to find the x-intercepts or zeroes, allowed easy graphing of the parabola. Today, the method of factoring by completing the square is introduced. Some equations have obvious square root solutions: $r^2 = 16$ and r = +/-4. Other equations have square root solutions that are not too obvious: Example: $a^2 – 6a + 9 = 25$ 25 obviously has a sq. root $(a – 3)^2 = 25$ and taking sq.rt. of both sides: $(a – 3) = \pm\sqrt{25}$ = ∀5 which produces two solutions: a – 3 = 5 or a = 8; and a – 3 = -5 or a = -2 U-DO: Put the quadratic equation: $a^2 – 6a + 9 = 25$ into vertex form and plot the resulting parabola. Is the vertex a maximum or minimum?  Answer: The vertex is a minimum. The vertex form $y = a(x – h)^2 + k$ where the coordinates of the vertex are at (h, k), of : $a^2 – 6a + 9 = 25$ is : $y = (a – 3)^2 - 25$ and h = 3 and k = -25.
Application: (10 minutes approx.)	Designing lampshades for manufacturing: Some lampshades have parabolic shapes. A imaginary company makes parabolic lampshades in various sizes. The engineer came up with a formula that described

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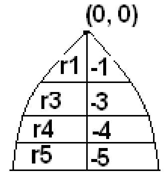
the radius of the bottom of a lampshade given the height of the lampshade. The relationship is: $h = -0.2r^2$ where h is height and r is radius. A drawing helps to show what is being described:



The heights are -1, -3, -4, and –5, because they are referenced to the vertex of (0, 0) and the radii are r1, r3, r4 and r5. In order to solve the formula for the various radii, the equation has to be first manipulated to have the radius squared on one side and then take the square root of both sides. In this example the negative values of the height are removed for simplicity, but in actuality, if you wanted an accurate mathematical representation of the parabola, then you would keep the negatives in the equation. Some applications can be adjusted as long as you know what you are adjusting and how it effects the results. In this case, we are only interested in the radii, so removing the negative is harmless. Solving: $h = 0.2r^2$ and $r^2 = h/0.2$ and then take the sq. root: $r = \pm\sqrt{\frac{h}{0.2}}$. At 1 inch from the vertex, the radius is $\pm\sqrt{\frac{1}{0.2}} = \pm\sqrt{\frac{10}{2}} = \pm\sqrt{5}$ or about 2.25 inches. Find the radii for heights of 3, 4, and 5 inches. Answer: for 3” : $r = \pm\sqrt{15}$; for 4”: $r = \pm\sqrt{20} = \pm 2\sqrt{5}$; and for 5”: $r = \pm 5$.
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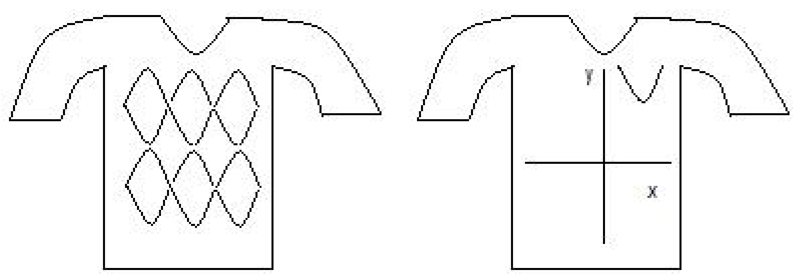
<tr><td>Direct Instruction and Practice (15 minutes approx.)</td><td>
Designing clothing patterns: A pattern for a T-shirt manufacturer is a parabolic pattern :



The first picture shows the pattern, the second shows a T-shirt with an x-y axis that can be used to position the pattern with computer graphics. This can be done fairly easily using what we have learned about quadratics.

The first parabola is defined by the equation: $y = x^2 - 6x + 13$, with the x values limited to a range. Can any of the students think of a way to keep the same shape and size of parabola, but move it around in a controlled manner within the x-y grid on the t-shirt? We would also like to rotate it and move it.

The answer is by using the vertex form of quadratic equation that describes the x-y position of the vertex, as well as the shape and size. The vertex form is:

$y = a(x - h)^2 + k$; where h is the x value and k is the y value of the vertex. The variable ‘a’ indicates the direction the parabola opens. If it is positive, it opens
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	upward, negative means opening downward. First convert the parabola from the design from standard form to vertex form: $y = x^2 - 6x + 13 = a(x - h)^2 + k$. In order to do this, we will employ a technique called "completing the square." The vertex form has a squared term: $(x - h)^2$, so if we can manipulate the equation to produce the squared portion, we can have a nice clean vertex form. To complete the square, take the terms from the quadratic that have the variable 'x' in them and try to come up with a squared term that it would equal by adding or subtracting numbers. Remember, we have the 'k' term that is separate from the squared term, so we can use that to offset any number needed to complete the square. $x^2 - 6x + ? = (x + ?)^2$ is our dilemma. One way to solve this is by trial and error. Another is using the reverse foil method and factoring, we know that the outer and inner terms must add up to -6 and since they are the same number (we are looking only for a solution that is a perfect square), it must be half the coefficient, and the formula $? = (6/2)^2$ can be used. Solving, we find 9 is the number that will complete the square: $x^2 - 6x + 9 = (x - 3)^2$. Wait a second, we are not quite done yet. The original parabola was defined by the equation: $y = x^2 - 6x + 13$ and we have $y = x^2 - 6x + 9$. What can be done to make the two equivalent? Answer is to just add 4 after the 9. $y = x^2 - 6x + 13 = x^2 - 6x + 9 + 4 = (x - 3)^2 + 4$; and it is in vertex form! Now, using your graphing calculator, put in this formula into the Y_1 of the 'Y=' function and graph it, adjusting the window size to display the T-shirt pattern. Recall how the vertex form describes the position of the vertex, have the students put in different formulas for Y_2 and Y_3 and Y_4 and so on to create an artistic pattern for the t-shirt. Here are some suggested equations: $Y_1 = (x - 3)^2 + 4$ $Y_2 = 1(x - 3)^2 - 4$ $Y_3 = (x + 3)^2 + 4$ $Y_4 = (x + 3)^2 - 4$ $Y_5 = (x - 3)^2 + 2$ $Y_6 = (x - 3)^2 - 2$ $Y_7 = (x + 3)^2 + 2$ $Y_8 = (x + 3)^2 - 2$
Practice: (10 minutes approx.)	U-DO: Practice completing the square on these equations: (Hint: $(b/2)^2$) 1. $x^2 + 8x + c$ (find c) Answer: c = 16 2. $x^2 - 4x + c$ (find c) Answer: c = 4 3. Solve for the square roots of both sides of this equation: $x^2 + 10x + 25 = 16$ Answer: $(x + 5)^2 = 16$; $(x + 5) = \pm 4$ 4. Solve for the square roots of both sides of this equation: $x^2 - 24x + 144 = 49$ Answer: $(x - 12)^2 = 49$; $(x - 12) = \pm 7$
Wrap-up	Wrap up closing comments and housekeeping.

and homework: (5 minutes approx.)	1. Solve for x by completing the square and taking the square root of both sides of the equation: $x^2 + 8x + 8 = 0$ Answer: $x^2 + 8x + 8 - 8 = 0 - 8$ then complete square $(8/2)^2 = 16$ and therefore: $x^2 + 8x + 16 = -8 + 16 = 8 = (x + 4)^2$; take sq.rt. of both sides results in: $x + 4 = \pm\sqrt{8} = \pm\sqrt{4}\sqrt{2} = \pm 2\sqrt{2}$ and then subtract 4 from both sides gives us the answer of $x = \pm 2\sqrt{2} - 4$