M^2=Math Mediator Lesson 31: Complex Numbers

Total Recall (Warm-up) (5 minutes approx.)

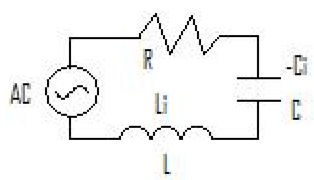
Total Recall: Exercise from yesterday's lesson on Square Roots.

1. If $t^2 = \sqrt{\frac{121}{144}}$ what does t = ? Answer: 11/12, take sq. root numerator and denominator.
2. $t^2 + 34 = 97$; t = ? Answer: $t = \sqrt{63} = \sqrt{9}\sqrt{7} = 3\sqrt{7}$
3. Simplify: $\frac{1}{5-\sqrt{6}}$ Answer: $\frac{1}{5-\sqrt{6}} \cdot \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{5+\sqrt{6}}{25-6} = \frac{5+\sqrt{6}}{19}$

Direct Instruction (15 minutes approx.)

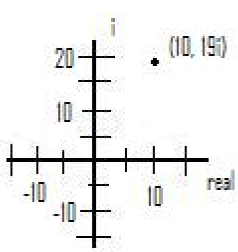
Algebra II. CA STD: 6.0

Electronics: Three components with a power source in an electronic circuit are analyzed using complex number 'i' to indicate phase shift, or an added attribute to consider on top of impedance. All three components have impedance, but only the inductor, L, and capacitor, C, have phase shift characteristics. The resistor, R, does not cause phase shift.



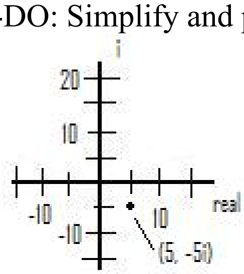
Component	Symbol	Impedance
Resistor	R	R
Inductor	L	Li
Capacitor	C	-Ci

Given that R = 10; C = 1; and L = 20; the total impedance of this series circuit would be: 10 + 20i – 1i = 10 + 19i (notice that the 'i' terms can be combined). Furthermore, this combination of real (10) and imaginary or complex (19i) can be plotted on a x-y Cartesian type of graph:



i = imaginary or complex unit = $\sqrt{-1}$; $i^2 = -1$.

U-DO: Simplify and plot: 15 + 5i – 10 – 10i ; Answer: 5 – 5i



Direct Instruction: (10 minutes approx.)

Vector Math: Complex numbers are used in spatial movements. A vector is a description of a movement; providing information about direction and distance. The imaginary number: 3 + 2i can be a vector, as well as a point, as we just described previously. In order to apply the vector, you must have a starting point.

If none is given, assume that (0, 0) is the starting point.

Activity: Mark out a real-complex coordinate system on the floor, with divisions from -10 to 10 on the real and -10i to 10i on the complex axis. Demonstrate moving from (0, 0) using the vector 2 + 2i. To do this, mark the start point and the end point of (2, 2i) and walk along the line from (0, 0) to (2, 2i). Vectors describe direction and distance. In our case, 2 + 2i is a vector with rectangular coordinates. That is the 2 is along one side of a rectangle, and the 2i is along another side of the rectangle. Vectors can also be described with polar coordinates. Can anyone guess how that might look, using the rectangular 2 + 2i vector, describing it with polar coordinates? Think about direction and distance…. Direction can be described with degrees, and distance with real numbers. In the case of 2 + 2i, the direction would be half of 90º, which is 45º, and the distance is found using the Pythagoran Theorem: because we are solving for the hypotenuse of a right triangle with sides of 2 and 2. The distance is the square root of the sum of the squares of the sides or :

$$\sqrt{2^2 + 2^2} = \sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$$

Question: What would be the vector needed to take them back to (0, 0) from the point of 2 + 2i? Answer: do the math: 2 + 2i +(a + bi) = 0 + 0i. <u>In order for complex vectors or numbers to be equal, then the real numbers must be equal and the coefficient of the complex numbers must be equal.</u> Therefore, 2 + a = 0 and 2 + b = 0 and both a and b must equal -2. The vector taking them back to zero is as you might have guessed: -2 – 2i.

Vectors are used in many different applications. In aviation, pilots and navigators need to combine vectors of aircraft speed and direction with wind speed and direction to get them to their destinations. In warfare, ballistics, cannons must account for wind speed and projectile speed. In ocean navigation there are currents that have offsetting vectors that must be accounted for when steering a ship.

U-DO: Solve for a and b in the following vector math problem:

3 + 2i + (a + bi) = 0 + 5i Answer: 3 + a = 0; a = -3; 2 + b = 5; b = 3

Direct Instruction and Practice (10 minutes approx.)

Multiplying Complex Numbers:

In vector multiplication, you obtain the product of the magnitude or distance of the vector, but you only get the sum of the angles of the vectors. Vector multiplication uses the FOIL method:

Example (3 + 2i)(-3 + 3i) = -9 – 6i + 9i + $6i^2$ = -9 + 3i + 6(-1) = -15 + 3i

Verify that the resulting vector has the product of the magnitudes and sum of angles of the vectors. Use the R>PΘ calculator function under "ANGLE" with rectangular coordinates of (3,2) and (-3,3) as arguments.

Vector 3 + 2i has a magnitude of $\sqrt{13}$ and direction of 33.69º

Vector -3 + 3i has a magnitude of $3\sqrt{2}$ and direction of 135º

Product Vector -15 + 3i has a magnitude of $\sqrt{234} = \sqrt{9}\sqrt{26} = 3\sqrt{26}$ and a

	direction of 168.69º. Note that $\sqrt{13}\cdot 3\sqrt{2} = 3\sqrt{26}$ and 33.69º + 135º = 168.69º This is the beauty of using complex numbers for vector multiplication. If we had used regular coordinates and multiplied them, the resulting vector direction would not have been the sum. We would have ended up with some y^2 term instead of i^2 or -1. Many computer software is written using complex notation for showing 3-D images and rotating those images. Fashion, mechanical display, packaging, 3-D modeling of all sorts, and computer games all rely on complex notation. Special 3-D complex number vectors, called "quaternions" solved "gimbal lock" in some early flight navigation systems. The gimbal is a yaw/pitch/roll sensing device in an aircraft and it would become useless, or lock up, in a dive or vertical assent. U-DO: Multiply the following vectors: 1. 5i(3 + 2i) Answer: $15i + 10i^2 = 15i - 10$ 2. (7 – 2i)(-3 + 4i) Answer: $-21 + 28i + 6i - 8i^2 = -13 + 34i$
Direct Instruction; practice and assessment: (10 minutes approx.)	Simplify complex quotients using complex conjugates: We prefer to not have any complex numbers in the denominator (that includes square roots as we mentioned before). Example: $\frac{3+2i}{5-3i}\cdot\frac{5+3i}{5+3i} = \frac{15+9i+10i+6i^2}{25+15i-15i-9i^2} = \frac{9+19i}{34}$; $\frac{5+3i}{5+3i}$ is the identity conjugate of the denominator 5 – 3i term. U-DO: 1. Simplify $\frac{4+3i}{2+3i}$ Answer: $\frac{4+3i}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{8-12i+6i-9i^2}{4-9i^2} = \frac{17-6i}{13}$ 2. Find the direction and distance of 2 + 5i. Answer: Use the R>PΘ calculator function under "ANGLE" with rectangular coordinates of (2,5) as argument to find direction in degrees. Use R>Pr for distance or use Pythagoran, sq. root of product of squares of sides = $\sqrt{29}$. Angle is 68.2º.
Wrap-up (5 minutes approx.)	Wrap up closing comments and housekeeping. Remember: i = imaginary or complex unit = $\sqrt{-1}$; $i^2 = -1$.