M^2 = Math Mediator Lesson 30: Sq. Roots & Quadratics

Total Recall (Warm-up) (5 minutes approx.)	Total Recall: Exercises from yesterday's lesson 1. Solve by factoring: $14t^2 - 21t = 0$; Answer: $7t(2t - 3) = 0$; $t = 0, 3/2$ 2. Solve by factoring: $16x^2 + 8x + 1 = 0$; Answer: $(4x + 1)(4x + 1)$; $x = -¼$ 3. Solve by factoring: $6s^2 - 7s - 5 = 0$; Answer: $(3x - 5)(2x + 1)$; $x = 5/3, -1/2$
Direct Instruction (15 minutes approx.) **CA Std 8.0**	Designing products to be drop-proof: modeling with quadratics. Good engineers design hand held products to be drop-proof, that is their housing and internal components will withstand the impact from dropping the device. I'm sure most of the students have dropped their cell phones or mp3 players. The energy in a dropped device is represented by the formula: $E = \frac{1}{2}mv^2$ and is also called kinetic energy, or energy due to its motion. The energy E is measured in joules, mass m is in kilograms, and velocity v is in meters/second. Plastic materials are rated for impact energy, and one particular choice of material is rated at 18 Joules. A typical hand held device has a mass of 0.16 kg (160 grams). Will this plastic material withstand a drop and protect this device? First, using the formula, we will solve for $v^2 = (2E)/m = 2(18)/.16 = 36/0.16$ and taking the **<u>square root</u>** of the numerator and the denominator: $v = 6/0.4 = 15$ m/s. Therefore, if our handheld device reaches a velocity of greater than 15 m/s and hits something hard, it will probably break. Gravity, as we have learned, is -9.8 m/s^2 (in the height formulas, we have been using -4.9 m/s^2 because the object is initially at rest and the average of 0 and gravity is ½ gravity), and velocity is m/s, so if there is no initial velocity (just let it fall from hand, don't throw it down on the ground), we can do some quick checks on velocity: Rounding -9.8 to -10 for quick check times 1 sec = -10 m/s and at 2 seconds it is -20 m/s. So, in order to reach -15 m/s, our critical velocity, the object needs to fall for about 1.5 seconds. Have a student take an erasure and drop it from head height and see if it is over one second. It is close to one second. In order to find the exact time use the height formula and a typical height of 6 feet or 2 meters (approximating). $h = -5t^2 + v_0t + h_0$ and since v_0 is zero, that term drops out and we have $h = -5t^2 + h_0$ and since h_0 is 2 and $h = 0$, we can solve for t in the equation: $0 = -5t^2 + 2$ and we find that by taking the **<u>square root</u>**: $t = +/-0.63$ seconds (plus or minus because of square roots can be either plus or minus mathematically, but with respect to time, negative does not make sense). Therefore, the object will only fall for about 0.63 seconds, which is less than 1 second and a velocity of -10 m/s, so it will not break. The 18 Joule plastic will be fine. In this typical engineering exercise, square roots were used twice, and needed to solve a quadratic equation. There will be more opportunity to use square roots in solving quadratics.
Properties of Square	Properties of Square Roots:

Roots and Practice (10 minutes approx.)	$\sqrt{x}$ is called square root of x or rad x, short for radical sign. x is the variable under the radical sign and is called the radicand. Variable 'a' is the square root of 'b' if $a^2 = b$. $\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$ or $\sqrt{45} = \sqrt{9}\cdot\sqrt{5} = 3\sqrt{5}$ (Simplifying the sq. root) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ or $\sqrt{\frac{36}{16}} = \frac{\sqrt{36}}{\sqrt{16}} = \frac{6}{4} = \frac{3}{2}$ Simplifying the square roots involves: 1. Removing all perfect squares from radicand or factors of radicand 2. Remove all radicals from the denominator. U-DO: Simplify the following radicals: 1. $\sqrt{18}$ Answer: $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$ 2. $\sqrt{\frac{64}{49}}$ Answer: $\sqrt{\frac{64}{49}} = \frac{\sqrt{64}}{\sqrt{49}} = \frac{8}{7}$ 3. $\sqrt{8}$ Answer: $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$
Direct Instruction and Practice (10 minutes approx.)	Removing or Rationalizing the denominator: In the case of this radical: $\sqrt{\frac{25}{3}}$ we can use the method we learned and take the square root of the numerator and denominator separately. The numerator is okay, but the denominator is rad 3, which cannot be simplified and leaves us with a radical in the denominator. This is not preferred. In any mathematical problem we know that 1 is the multiplicative identity and multiplying one by any number results in the number remaining the same. The key here is that 1 can take on many forms, such as 3/3 or $\frac{\sqrt{3}}{\sqrt{3}}$. Multiplying our original problem by 1: $\frac{\sqrt{25}}{\sqrt{3}} = \frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ which is the preferred simplified form. U-DO: Simplify the following: 1. $\sqrt{\frac{16}{5}}$ Answer: $\frac{\sqrt{16}}{\sqrt{5}} = \frac{4}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$ 2 $\frac{4}{\sqrt{3}}$ Answer: $\frac{4}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ 3. $\frac{3}{2+\sqrt{2}}$ Answer: $\frac{3}{2+\sqrt{2}} = \frac{3}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{6-3\sqrt{2}}{4-2} = \frac{6-3\sqrt{2}}{2}$

	4. $\frac{7}{1-\sqrt{3}}$ Answer: $\frac{7}{1-\sqrt{3}} = \frac{7}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{7+7\sqrt{3}}{1-3} = \frac{7-7\sqrt{3}}{-2}$
Direct Instruction: (10 minutes approx.)	Solving Quadratics using square roots: From earlier example: $t^2 = 2/5$, and $t = \pm\sqrt{\frac{2}{5}} = \pm 0.63$ Another example: $-5t^2 + 50 = 10$; solve for t: $-5t^2 = -40$; $t^2 = 40/5 = 8$ and taking the sq. root of both sides: $t = \pm\sqrt{8} = \pm\sqrt{4}\sqrt{2} = \pm 2\sqrt{2}$ U-DO: Solve the following quadratics: 1. $-5t^2 + 17 = 2$ Answer: $-5t^2 = -15$; $t^2 = 3$; and $t = \pm\sqrt{3}$ 2. $x^2 - 7 = 20$ Answer: $x^2 = 27$; $x = \pm\sqrt{27} = \pm\sqrt{9}\sqrt{3} = \pm 3\sqrt{2}$
Wrap-up (5 minutes approx.)	Wrap up closing comments and housekeeping. Homework: Each year the engineering students at universities go through a ritual of dropping eggs, watermelons, etc.. off the roof of a tall building, about 20 meters tall. Find the time when an object will hit the ground from 20 meters and the energy a 60 gram object has. Use the height formula: $h = -5t^2 + v_0t + h_0$ and energy formula: $E = (1/2)mv^2$ (remember m is in kilograms). Answer: time = 2 seconds; energy is 12 Joules.