M^2=Math Mediator Lesson 23: Multiplying Matrices

Total Recall (Warm-up) (7 minutes approx.)	Total Recall: Exercise from yesterday's lesson on Matrix Operations. $A=\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}$; $B=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}$; $C=\begin{bmatrix}-3 & 2\\ 4 & 2\end{bmatrix}$; Find the following: 1. A + B ? Answer: $A+B=\begin{bmatrix}3 & 1\\ 5 & 7\end{bmatrix}$ 2. B – C ? Answer: $B-C=\begin{bmatrix}4 & -4\\ -2 & 1\end{bmatrix}$ 3. 2A – 3B = ? Answer: $2A-3B=\begin{bmatrix}1 & 12\\ 0 & -1\end{bmatrix}$ 4. What are all of these matrix's dimensions? 2x2
Direct Instruction (10 minutes approx.)	In order to multiply one matrix times another, the number of columns of the first matrix must equal the number of rows in the second. To multiply Matrix A ($rows_A$ x $columns_A$) x Matrix B ($rows_B$ x $columns_B$), then $columns_A$ = $rows_B$ must be true. The resulting Matrix from multiplying Matrix A times Matrix B has the dimensions of $rows_A$ by $columns_B$. $A=\begin{bmatrix}2 & 1 & 4\\ 3 & 5 & 3\end{bmatrix}$; $B=\begin{bmatrix}2 & 3\\ 1 & 4\\ 5 & 2\end{bmatrix}$; To multiply, first check the dimensions to see if these two matrices can be multiplied. Does the columns from A equal the rows from B? Yes, Matrix A is 2 x 3 and Matrix B is 3 x 2. What is going to be the resulting matrix? Answer is rows from A (2) by columns from B (2) or 2x2. If Matrix F is 2 x 5 and Matrix G is 5 x 3, what would be the resulting matrix dimension if the two were multiplied? Answer is 2x3, because they can be multiplied, since columns F equals rows G, each 5. If Matrix N is 3 x 4 and Matrix P is 3 x 3, what would be the resulting matrix dimension if the two were multiplied? They cannot be multiplied since the columns of N do not equal the rows of P. To multiply matrix A and B from above, the technique is to take the first row from matrix A and rotate it clockwise 90º and match the number up with the first column of matrix B: $A=\begin{bmatrix}2 & 1 & 4\\ 3 & 5 & 3\end{bmatrix}$; $B=\begin{bmatrix}2 & 3\\ 1 & 4\\ 5 & 2\end{bmatrix}$ $A\cdot B=\begin{bmatrix}2\cdot2+1\cdot1+4\cdot5 & 2\cdot3+1\cdot4+4\cdot2\\ 3\cdot2+5\cdot1+3\cdot5 & 3\cdot3+5\cdot4+3\cdot2\end{bmatrix}$

	$A \cdot B = \begin{bmatrix} 4+1+20 & 6+4+8 \\ 6+5+15 & 9+20+6 \end{bmatrix} = \begin{bmatrix} 25 & 18 \\ 26 & 35 \end{bmatrix}$
Review: Professor of the day! (10 minutes approx.)	Ask if any student wants to be professor of the day and explain how the following matrices would be multiplied, if they can? $C = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$; $D = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ Answer: Both are 2x2, so they can be multiplied and the resulting matrix is 2x2. $C \cdot D = \begin{bmatrix} 2 \cdot 3 + (-2) \cdot 2 & 2 \cdot 4 + (-2) \cdot 1 \\ 3 \cdot 3 + 1 \cdot 2 & 3 \cdot 4 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6-4 & 8-2 \\ 9+2 & 12+1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 11 & 13 \end{bmatrix}$
Practice and assessment: (10 minutes approx.)	U-DO: Use the following matrices: $R = \begin{bmatrix} 2 & 7 \\ 5 & -1 \\ 4 & 3 \end{bmatrix}$; $S = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 4 & 2 \end{bmatrix}$; $T = \begin{bmatrix} 1 & 2 & 2 \\ 5 & 4 & 3 \end{bmatrix}$ 1. Prove or disprove that the distributive property works with matrices. (Hint: distributive: R(S + T) = RS + RT). Answer: yes it works. 2. Does the commutative property work? Find SR. Does it equal RS? Ans=NO.
Direct Instruction; properties of matrix multiplication: (8 minutes approx.)	Properties of Matrix Multiplication: Given matrices A, B and C. 1. Associative: A(BC) = (AB)C 2. Left Distributive: A(B + C) = AB + AC 3. Right Distributive: (B + C)A = BA + CA 4. NOT Commutative: AB ≠ BA 5. Associative with scalar(k): k(AB) = (kA)B = A(kB)
Wrap-up and assignment (10 minutes approx.)	Wrap up closing comments and housekeeping. Write a reply to the following request: Dear Dr. Math: The accounting department has informed me that my calculation is wrong, but did not explain where or why. Can you please help me to correct the mistake? Company: San Diego Examiner Newspaper Document: Cost Analysis for Yearly Equipment Budget Equipment: PC's Copiers Fax Writers: 5 3 2 Editors: 3 1 1 Advertising: 4 3 3 Maintainence and Labor for each in USD$: 400 200 100 The total budget for the year is calculated by multiplying each piece of

equipment times the M & L and showing as a charge for each group. To calculate each groups cost, I did this matrix multiplication:

$$Equipment = \begin{bmatrix} 5 & 3 & 2 \\ 3 & 1 & 1 \\ 4 & 3 & 3 \end{bmatrix}; \; M \;\&\; L = \begin{bmatrix} 400 \\ 200 \\ 100 \end{bmatrix};$$

$$Eq \times M \;\&\; L = \begin{bmatrix} 5\cdot 400 + 3\cdot 200 + 4\cdot 100 \\ 3\cdot 400 + 1\cdot 200 + 3\cdot 100 \\ 2\cdot 400 + 1\cdot 200 + 3\cdot 100 \end{bmatrix} = \begin{bmatrix} 3000 \\ 1700 \\ 1300 \end{bmatrix}$$

Totals per group: Writers: $3000
Editors: $1700
Advertising: $1300

Where did I make my mistake?

Sincerely,
Pablo Publisher

ANSWER: The mistake is in the multiplication method. Pablo did not take the rows from the first matrix to rotate and match up with the columns. Pablo mistakenly took the columns and matched them up to the column. The correct method looks like this:

$$Eq \times M \;\&\; L = \begin{bmatrix} 5\cdot 400 + 3\cdot 200 + 2\cdot 100 \\ 3\cdot 400 + 1\cdot 200 + 1\cdot 100 \\ 4\cdot 400 + 3\cdot 200 + 3\cdot 100 \end{bmatrix} = \begin{bmatrix} 2800 \\ 1500 \\ 2500 \end{bmatrix}$$

Totals per group: Writers: $2800
Editors: $1500
Advertising: $2500