

# M<sup>2</sup>=Math Mediator Lesson 22: Matrix Operations

<p>Total Recall (Warm-up) (5 minutes approx.)</p>	<p>Total Recall: Exercise from yesterday's lesson on Matrices.</p> <ol style="list-style-type: none"> <li>What are the dimensions of the following matrix? Ans: 3x4</li> <li>What is the element at row 3, column 2? Ans: 5             <p>a. <math>\begin{bmatrix} 1 &amp; 4 &amp; 7 &amp; 8 \\ 3 &amp; 2 &amp; 6 &amp; 4 \\ 1 &amp; 5 &amp; 9 &amp; 3 \end{bmatrix}</math></p> </li> <li>What are the special names for the following matrices?             <p>a. <math>\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}</math>; Ans: column matrix, zero matrix</p> <p>b. <math>\begin{bmatrix} 1 &amp; 2 \\ 3 &amp; 4 \end{bmatrix}</math>; Ans: square matrix.</p> </li> <li>Perform the matrix operation:             <p>a. <math>\begin{bmatrix} 4 &amp; 2 \\ 3 &amp; 6 \end{bmatrix} + \begin{bmatrix} 1 &amp; 3 \\ 5 &amp; 2 \end{bmatrix}</math>; Ans: <math>\begin{bmatrix} 4+1 &amp; 2+3 \\ 3+5 &amp; 6+2 \end{bmatrix} = \begin{bmatrix} 5 &amp; 5 \\ 8 &amp; 8 \end{bmatrix}</math></p> <p>b. <math>\begin{bmatrix} 2x &amp; 3 &amp; 4y \end{bmatrix} - \begin{bmatrix} 4 \\ 3y \end{bmatrix}</math>; Ans: Impossible, not same dimensions</p> </li> </ol>
<p>Direct Instruction (8 minutes approx.)</p>	<p>Since matrices can be added and subtracted if they are of the same dimension, we can then solve for variables in matrix operations:</p> <p>EXAMPLE: <math>\begin{bmatrix} 2x &amp; 4 \\ 3 &amp; 21y \end{bmatrix} - \begin{bmatrix} 4 &amp; 4 \\ 2 &amp; 5 \end{bmatrix} = \begin{bmatrix} 4 &amp; 0 \\ 1 &amp; 2 \end{bmatrix}</math> solve for x and y.</p> <p>Solution: Using matrix subtraction and the given example: <math>2x - 4 = 4</math>; <math>x = 4</math> and <math>21y - 5 = 2</math>; <math>y = 1/3</math>.</p> <p>U-DO: Solve for a and b in the following matrix operation:  <math>\begin{bmatrix} 40a &amp; 16 &amp; 25b \end{bmatrix} + \begin{bmatrix} 25a &amp; 4 &amp; 5 \end{bmatrix} = \begin{bmatrix} 13 &amp; 20 &amp; -20 \end{bmatrix}</math>; Answer: <math>a = 1/5</math>; <math>b = -1</math></p>
<p>Direct Instruction and practice (7 minutes approx.)</p>	<p>Matrices can also be multiplied by numbers, or scaled.</p> <p>Example: <math>3 \begin{bmatrix} 4 &amp; 5 &amp; 3 \\ 2 &amp; 1 &amp; 2 \\ 3 &amp; 2 &amp; 4 \end{bmatrix} = \begin{bmatrix} 3 \cdot 4 &amp; 3 \cdot 5 &amp; 3 \cdot 3 \\ 3 \cdot 2 &amp; 3 \cdot 1 &amp; 3 \cdot 2 \\ 3 \cdot 3 &amp; 3 \cdot 2 &amp; 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 12 &amp; 15 &amp; 9 \\ 6 &amp; 3 &amp; 6 \\ 9 &amp; 6 &amp; 12 \end{bmatrix}</math></p> <p>U-DO: Solve the scaled matrix operation:</p> <ol style="list-style-type: none"> <li><math>0.5 \begin{bmatrix} 20 &amp; 120 \\ 12 &amp; 44 \end{bmatrix}</math>; Answer: <math>\begin{bmatrix} 10 &amp; 60 \\ 6 &amp; 22 \end{bmatrix}</math></li> </ol>

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	<p>2. <math>-2 \begin{bmatrix} 2 &amp; 1 \\ 4 &amp; 0 \end{bmatrix} - 3 \begin{bmatrix} -2 &amp; 1 \\ 1 &amp; 5 \end{bmatrix}</math>; Answer: <math>\begin{bmatrix} 2 &amp; -5 \\ -11 &amp; -15 \end{bmatrix}</math></p> <p>Extra: <math>\frac{1}{4} \left( \begin{bmatrix} 3 &amp; 1 \\ -2 &amp; 0 \end{bmatrix} + \begin{bmatrix} 9 &amp; 7 \\ -2 &amp; -4 \end{bmatrix} \right)</math>; Answer: <math>\begin{bmatrix} 3 &amp; 2 \\ -1 &amp; -1 \end{bmatrix}</math></p>																																			
<p><b>Direct Instruction and Practice</b> (10 minutes approx.)</p>	<p>The extra problem in the above section can be solved one of two ways, because matrix operations are distributive with respect to addition and subtraction. That is to say that for same dimension Matrix A and Matrix B: <math>k(A+B) = kA + kB</math> and <math>k(A-B) = kA - kB</math>. The best solution can often be chosen by a quick analysis of how clean a multiplication or division by the scalar is on either the individual matrices or the resultant matrix.</p> <p>Challenge: Use matrix math to find the average monthly sales number for the following corporation's sales of October and November:</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">Month:</th> <th colspan="3" style="text-align: center;">October</th> <th colspan="3" style="text-align: center;">November</th> </tr> <tr> <th style="text-align: left;">Sizes:</th> <th style="text-align: center;">petite</th> <th style="text-align: center;">medium</th> <th style="text-align: center;">large</th> <th style="text-align: center;">petite</th> <th style="text-align: center;">medium</th> <th style="text-align: center;">large</th> </tr> </thead> <tbody> <tr> <td>T-shirts:</td> <td style="text-align: center;">40</td> <td style="text-align: center;">45</td> <td style="text-align: center;">43</td> <td style="text-align: center;">36</td> <td style="text-align: center;">41</td> <td style="text-align: center;">49</td> </tr> <tr> <td>Button shirts:</td> <td style="text-align: center;">35</td> <td style="text-align: center;">65</td> <td style="text-align: center;">41</td> <td style="text-align: center;">48</td> <td style="text-align: center;">58</td> <td style="text-align: center;">52</td> </tr> <tr> <td>Polo shirts:</td> <td style="text-align: center;">46</td> <td style="text-align: center;">58</td> <td style="text-align: center;">49</td> <td style="text-align: center;">39</td> <td style="text-align: center;">49</td> <td style="text-align: center;">45</td> </tr> </tbody> </table> <p>ANSWER: Average is the sum divided by number of occurrences. Using October as Matrix A and November as Matrix B: <math>Avg = \frac{1}{2}(A + B)</math> or:</p> $\frac{1}{2} \left( \begin{bmatrix} 40 & 45 & 43 \\ 35 & 65 & 41 \\ 46 & 58 & 49 \end{bmatrix} + \begin{bmatrix} 36 & 41 & 49 \\ 47 & 57 & 53 \\ 38 & 48 & 45 \end{bmatrix} \right) = \begin{bmatrix} 38 & 43 & 46 \\ 41 & 61 & 47 \\ 42 & 53 & 47 \end{bmatrix}$	Month:	October			November			Sizes:	petite	medium	large	petite	medium	large	T-shirts:	40	45	43	36	41	49	Button shirts:	35	65	41	48	58	52	Polo shirts:	46	58	49	39	49	45
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<p><b>Practice and assessment:</b> (10 minutes approx.)</p>	<p>Find the average monthly hours spend on the various activities of these three youth. Then use the average hours to estimate the yearly hours spent on each activity.</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">Month:</th> <th colspan="3" style="text-align: center;">September</th> <th colspan="3" style="text-align: center;">October</th> </tr> <tr> <th style="text-align: left;">Activity:</th> <th style="text-align: center;">Movies</th> <th style="text-align: center;">Video Games</th> <th style="text-align: center;">Outdoor Games</th> <th style="text-align: center;">Movies</th> <th style="text-align: center;">Video Games</th> <th style="text-align: center;">Outdoor Games</th> </tr> </thead> <tbody> <tr> <td>Sarah:</td> <td style="text-align: center;">12</td> <td style="text-align: center;">8</td> <td style="text-align: center;">4</td> <td style="text-align: center;">8</td> <td style="text-align: center;">6</td> <td style="text-align: center;">4</td> </tr> <tr> <td>Roberto:</td> <td style="text-align: center;">8</td> <td style="text-align: center;">12</td> <td style="text-align: center;">8</td> <td style="text-align: center;">6</td> <td style="text-align: center;">24</td> <td style="text-align: center;">7</td> </tr> <tr> <td>Phil:</td> <td style="text-align: center;">9</td> <td style="text-align: center;">20</td> <td style="text-align: center;">5</td> <td style="text-align: center;">7</td> <td style="text-align: center;">22</td> <td style="text-align: center;">4</td> </tr> </tbody> </table> <p>Ans: <math>\frac{1}{2} \left( \begin{bmatrix} 12 &amp; 8 &amp; 4 \\ 8 &amp; 12 &amp; 8 \\ 9 &amp; 20 &amp; 5 \end{bmatrix} + \begin{bmatrix} 8 &amp; 6 &amp; 4 \\ 6 &amp; 24 &amp; 7 \\ 7 &amp; 22 &amp; 4 \end{bmatrix} \right) = \begin{bmatrix} 10 &amp; 7 &amp; 4 \\ 7 &amp; 18 &amp; 7.5 \\ 8 &amp; 21 &amp; 4.5 \end{bmatrix} \times 12 = \begin{bmatrix} 120 &amp; 84 &amp; 48 \\ 84 &amp; 216 &amp; 90 \\ 96 &amp; 252 &amp; 54 \end{bmatrix}</math></p>	Month:	September			October			Activity:	Movies	Video Games	Outdoor Games	Movies	Video Games	Outdoor Games	Sarah:	12	8	4	8	6	4	Roberto:	8	12	8	6	24	7	Phil:	9	20	5	7	22	4
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<p><b>Exercise:</b></p>	<p>Use the graphing calculator to perform Matrix operations: Go into the MATRIX</p>																																			

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Graphing Calculator (13 minutes approx.)	area of your graphing calculator by pressing '2 <sup>nd</sup> ' button and MATRIX. Edit the A and B matrix to September and October activity data from the above problem. From the MATRIX – NAMES, select 'A', press +, and select 'B' from MATRIX – NAMES. Press ENTER, then press ENTER again, which brings up ANS*, and press times 0.5, to obtain the average Matrix. It should be the same as the answer in the above problem. Try some other matrix operations if you have time. Use examples from our lesson today.
Wrap-up (2 minutes approx.)	Wrap up closing comments and housekeeping.