M^2=Math Mediator   Lesson 21: Introduction to Matrices

Intro: (5 minutes approx.)	From previous lessons on systems of equations, you have learned how to solve two simultaneous equations with two variables using the elimination or substitution methods. You have also seen how to use this method twice in solving systems of three equations with 3 variables. Next, we will cover methods to solve systems of equations involving 4 or more equations and variables. This technique is used in many fields of research and analysis. In genetics, researchers use multiple simultaneous equations to analyze various gene combinations. Champion horse breeders want to know the chances of success in breeding another triple crown race horse. In electronics, circuits are analyzed for loop current interaction and control using simultaneous equations. In the newest cell phone device, the manufacturer wants to make sure that the lowest amount of power is being used in order to maximize the battery life. In order to analyze and solve these types of multi-variable systems of equations, we will use the matrices method. That is what we will discuss today.
Direct Instruction (5 minutes approx.)	Consider this example of 4 equations and 4 variables: $2a + 3b + 4c + 5d = 110$ $a - b + 3c - 2d = 5$ $3a + 2b - c + 2d = 20$ $4a - 2b + 2c - 3d = 12$ Notice that all four of these equations is in Standard Form. That is the first terms are all 'a', next are all 'b' and so on, with the constant value always on the right side of the equal sign. With all equations in this form, we remove the coefficients from all the variables and make a matrix of the variable coefficients: $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & -1 & 3 & -2 \\ 3 & 2 & -1 & 2 \\ 4 & -2 & 2 & -3 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 110 \\ 5 \\ 20 \\ 12 \end{bmatrix}$ *Notice how minus signs have become negative values. 'A' Matrix x 'X' matrix = 'B' Matrix or: $A \cdot X = B$ using matrix math, we will create an inverse (A^{-1}) matrix and multiply that times the original A matrix to create a solution for the variable or X matrix. First we need to learn the basics of matrix terminology and construction.
Review (5 minutes approx.)	A Matrix is a rectangular array of variables, constants or some combination of them. Matrices are organized into rows and columns. Have the students stretch out their arms and hands sideways. These are the rows of data, they go across the page, like oars on a boat to "row" the boat. Arms and hands up in the air. These are columns of data, that go up and down a page, like columns in an ancient Greek temple. Matrices are enclosed in brackets with tops and bottoms of the enclosure angled in at 90 degrees [ ]. The elements in each row are related in

	some way, as are the elements in each column.
Practice (10 minutes approx.)	In researching colleges, the following theoretical information was discovered: SDSU: enrollment was 28,527, tuition was 3176 and room/board was 10904. Cal. St. San Marcos: tuition = 3092; enrollment = 7558; r/b = 9230 UCSD: r/b = 10,237; tuition = 13660; enrollment = 21369 USD: tuition = 32,300; r/b = 10,960; enrollment = 7504 Put this data in a 4 row by 3 column matrix. Answer: $\begin{matrix} & \underline{\text{Tuition}} & \underline{\text{r/b}} & \underline{\text{enrollment}} \\ SDSU & 3176 & 10904 & 28527 \\ CSSM & 3092 & 9230 & 7558 \\ USCSD & 13660 & 10237 & 21369 \\ USD & 32300 & 10960 & 7504 \end{matrix}$ What does this data tell us? SDSU costs lower, but has a huge population of students (large classroom size) USD has a small population, small classroom size, but costs big bucks. You pay for more contact with your professors.
Direct Instruction: (10 minutes approx.)	Matrices are said to have dimension with the convention of rows by columns. Rows are always the first number, columns the second. You can remember this by being in reverse alphabetical order or by some reference that helps you (I like RC cola for matrix refreshment). What is the dimension of the matrix in the college example? Answer: 4 x 3. What is the element that is in the 4,2 position? Answer: Row 4, column 2 has the number 10960. Other terminology: Row Matrix: A matrix with only one row Column Matrix: A matrix with only one column Square Matrix: A matrix with equal rows and columns Zero Matrix: A matrix will every element being zero. Equal Matrices: Matrices that have the same dimensions and have equal corresponding elements. Examples of equal matrices: $\begin{bmatrix} 6 & 3 \\ 0 & 9 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4+2 & 2+1 \\ 3-3 & 6+3 \\ 2-1 & 1+2 \end{bmatrix}; \begin{bmatrix} 5 & 6 & 0 \\ 0 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 0 \\ 0 & 7 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

	Not equal matrices: $\begin{bmatrix}1 & 2\\8 & 5\end{bmatrix} \neq \begin{bmatrix}1 & 8\\2 & 5\end{bmatrix}$ and $\begin{bmatrix}4 & 3\\8 & 2\\5 & 1\end{bmatrix} \neq \begin{bmatrix}5 & 8 & 4\\1 & 2 & 3\end{bmatrix}$ The first two are equal in dimension, but not equal elements in the same position. The next two matrices are rotated 90 degrees to each other, but do not have the same dimensions, so are not equal.
Exercise: (5 minutes approx.)	1. In matrix A = $\begin{bmatrix}3 & 4 & 1 & 7\\2 & 6 & 4 & 9\end{bmatrix}$; 7 is located in which row and column? Answer: 1,4 2. What is the dimension of the matrix in number 1 exercise? Answer: 2x4 3. Solve for the variables in the following matrices: a. $\begin{bmatrix}9x & 7y\end{bmatrix} = \begin{bmatrix}36 & 49\end{bmatrix}$; Answer: x=4; y=7 b. $\begin{bmatrix}2x & 3y\\3 & 14-z\end{bmatrix} = \begin{bmatrix}22 & 21\\21w & 7\end{bmatrix}$; Answer: w=1/7; x=11; y=7; z=7.
Instruction and Exercise: (10 minutes approx.)	Matrix Addition: If Matrix A = $\begin{bmatrix}6 & 8\\7 & 9\end{bmatrix}$ and matrix B = $\begin{bmatrix}2 & 4\\1 & 5\end{bmatrix}$; then the addition of Matrix A and Matrix B can only be done if the dimensions are the same and results in a matrix of the same dimension. The solution elements are the sum of the two elements with same location in the original matrices: A + B = $\begin{bmatrix}6+2 & 8+4\\7+1 & 9+5\end{bmatrix} = \begin{bmatrix}8 & 12\\8 & 14\end{bmatrix}$ U-DO: 1. Perform the indicated operation, if possible. a. $\begin{bmatrix}7 & -10\\-6 & 8\end{bmatrix} - \begin{bmatrix}4 & -15\\-2 & 3\end{bmatrix}$; Answer: $\begin{bmatrix}3 & 5\\-4 & 5\end{bmatrix}$ b. $\begin{bmatrix}3 & 4 & 7\end{bmatrix} + \begin{bmatrix}7\\2\\1\end{bmatrix}$; Answer: Impossible, not same dimensions c. $\begin{bmatrix}2 & -3 & 7\\7 & 4 & -5\\-4 & 6 & 8\end{bmatrix} - \begin{bmatrix}5 & 7 & 4\\3 & 5 & 1\\-9 & -2 & -5\end{bmatrix}$; Answer: $\begin{bmatrix}-3 & -10 & 3\\4 & -9 & -6\\5 & 8 & 13\end{bmatrix}$
Wrap-up	Wrap up closing comments and housekeeping.

and homework assignment (5 minutes approx.)	